XXXV. *Some New Properties in Conic Sections, discovered by* Edward Waring, *M. A.* Lucasian *Professor of the Mathematics in the University of* Cambridge, *and F R. S. to* Charles Morton, *M. D. Sec. R. S.*

THEOR. I.

Read June 21, 1764.

SIT ellipsis APBQCRDSET, &c. describantur circa eam duo polygona [TAB. XIII. Fig. 1.] (*abcdef*, &c. *pqrstv*, &c.) eundem laterum numerum habentia, & quorum latera ad respectiva contactuum puncta (APBQCRDS, &c.) in duas æquales partes dividuntur, i. e. $aA = Ab$, $bB = Bc$, $cC = Cd$, &c. $pP = Pq$, $qQ = Qr$, $rR = Rs$, &c. & erit summa quadratorum ex singulis unius polygoni lateribus æqualis summæ quadratorum ex singulis alterius polygoni lateribus, i. e.

$$ab^2 + bc^2 + cd^2 + de^2 + ef^2 +, \&c. = pq^2 + qr^2 + rs^2 + st^2 + tv^2 +, \&c.$$

Cor. Ducantur lineæ AB, BC, CD, DE, EF, &c. PQ, QR, RS, ST, TV, &c. & erit

$$AB^2 + BC^2 + CD^2 + DE^2 + EF^2 + \&c. = PQ^2 + QR^2 + RS^2 + ST^2 + TV^2 + \&c.$$

THEOR. II.

Iisdem positis sit O centrum ellipseos, & ducantur lineæ OA, OP, OB, OQ, OC, OR, OD, OS, &c. erit

$OA^2 + OB^2 + OC^2 + OD^2 + \&c. = OP^2 + OQ^2 + OR^2 + OS^2 + \&c.$

Cor. Ducantur etiam lineæ Oa, Op, Ob, Oq, Oc, Or, Od, Os, &c. & erit

$Oa^2 + Ob^2 + Oc^2 + Od^2 + \&c. = Op^2 + Oq^2 + Or^2 + Os^2 + \&c.$

Hæc etiam vera sunt de polygonis inter conjugatas hyperbolas eodem modo descriptis.

THEOR. III.

Sit conica sectio MPQRSTM′ &c. [Fig. 2.] cujus diameter sit AL, et ejus ordinata ML′; sit $Mp = Mv$, & consequenter $Lp = Lv$.

Ducantur lineæ pq, qr, rs, st, tv, &c. quæ respective tangant conicam sectionem in punctis P, Q, R, S, T, &c. & erit contentum

$pP \times qQ \times rR \times Ss \times \&c. = Pq \times Qr \times Rs \times St \times Tv \times \&c.$ vel, quod idem est, summa omnium hujus generis rationum ($Pp : Pq$, $Qq : Qr$, $Rr : Rs$, $Ss : St$, &c.) erit nihilo æqualis.

Cor. 1. Sit ellipsis PQRSTV &c. circa eam describatur quodcunque polygonum ($pqrstuw$, &c.), [Fig.

Fig. 1.

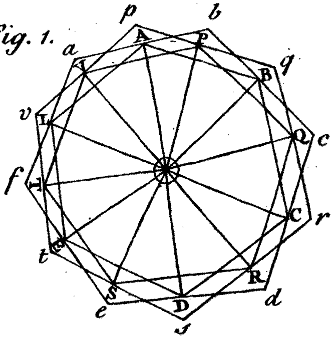


Fig. 2.

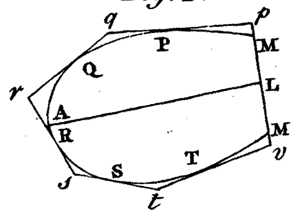


Fig. 3.

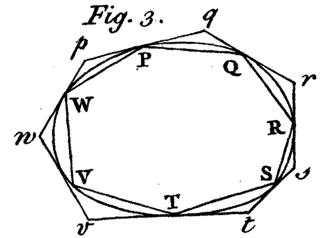


Fig. 4.

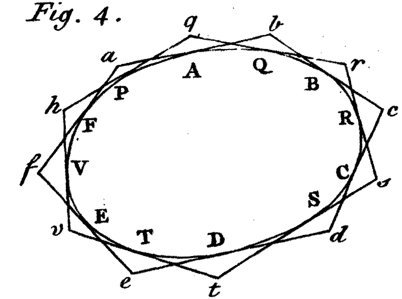


Fig. 5.

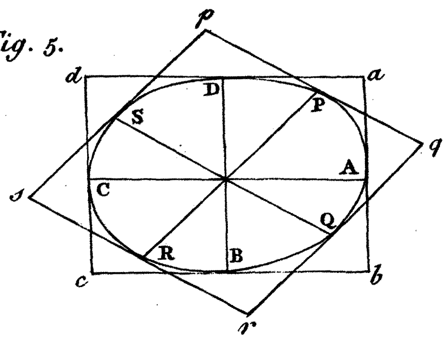


Fig. 6.

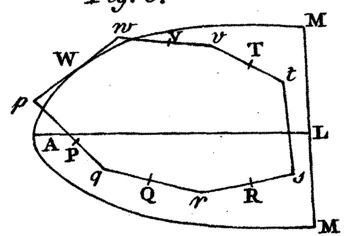
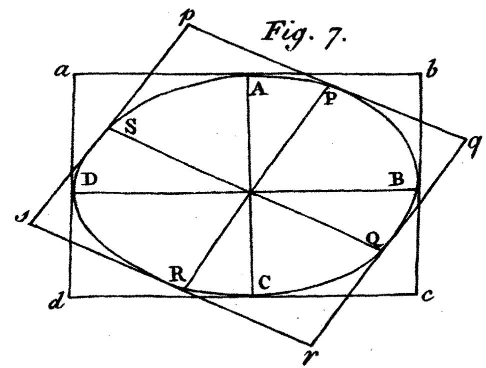


Fig. 7.



J. Mynde sc.

[Fig. 3.] cujus latera reſpective tangant ellipſim in punctis P, Q, R, S, T, V, &c. & erit contentum

$pP \times qQ \times rR \times sS \times tT \times vV$ &c. $= Pq \times Qr \times Rs \times St \times Tv \times Vw \times$ &c.

Cor. Ducantur lineæ PQ, QR, RS, ST, &c. & pro finibus angulorum WPp, QPq, RQr, QRr, SRs, TSt, &c. ſcribantur reſpective a, p, b, q, c, r, d, s, &c. & erit

$abcd$ &c. $= pqrs$ &c.

Et ſic de polygonis inter conjugatis hyperbolas inſcriptis.

Idem verum eſt de polygono, cujus laterum ſumma vel area minima ſit, circa quamcunque ovalem in ſeſe ſemper concavam deſcripto, ut conſtat e noſtra Miſcell. Anal.

THEOR. IV.

Sit ellipſis PAQBRCSDTEVF, &c. [Fig. 4.] circa eam deſcribantur duo polygona $abcdef$, &c. $pqrstu$, &c. eundem laterum numerum habentia; eorum latera ab, bc, cd, de, ef, &c. pq, qr, rs, st, tv, &c. reſpective tangant ellipſim in punctis A, B, C, D, E, F, &c. & P, Q, R, S, T, U, &c. & ſit $aA : Ab :: pP : Pq$, & $bB : Bc :: qQ : Qr$ & $cC : Cd :: rR : Rs$ & $dD : De :: sS : St$, & ſic deinceps. Et area polygoni $abcdef$, &c. æqualis erit areæ polygoni $pqrstv$, &c.

Cor. Duo parallelogramma ($abcd$ & $pqrs$) circa datæ ellipſeos conjugatas diametros (AC & BD; PR, QS) [Fig. 5.] deſcripta, erunt inter ſe æqualia.

In hoc casu enim $aA = Ab$, $bB = Bc$, $cC = Cd$, $dD = Da$, & $pP = Pq$, $qQ = Qr$, $rR = Rs$, $sS = Sp$; & consequenter $aA : Ab :: pP : Pq$ & $bB : Bc :: qQ : Qr$, & sic deinceps: ergo per theorema hæc duo parallelogramma erunt inter se æqualia, quæ est notissima ellipseos proprietas.

Idem dici potest de polygonis inter conjugatas hyperbolas eodem modo descriptis.

THEOR. V.

Rotetur conica sectio circa diametrum ejus (AL), & sit MAM', &c. solidum exinde generatum; sint pq, qr, rs, st, tv, vw, wp, &c. [Fig. 6.] lineæ, quæ tangant solidum in respectivis punctis P, Q, R, S, T, V, W, &c. & erit contentum

$$pP \times qQ \times rR \times sS \times tT \times vV \times wW \times \&c. = Pq \times Qr \times Rs \times St \times Tv \times Vw \times \&c.$$

THEOR. VI.

Sit ellipsis $APBQCR$, &c. rotetur circa diametrum ejus BD; & circa conjugatas diametros (AC & BD, PR & QS) describantur elliptici cylindri ($pqrs$ & $acbd$) [Fig. 7.] solidum generatum circumscribentes, & erunt hi duo cylindri inter se æquales.

Sint duo solida e truncatis conis composita, solidum generatum circumscribentibus, & quorum latera continuo

tinuo eâdem ratione ad puncta contactuum dividuntur; erunt hæc duo solida inter se æqualia.

Et sic de solidis inter conjugatas hyperboloides eodem modo descriptis.

Facile constant plures consimiles conicarum sectionum proprietates.

Hujus generis proprietates affirmari possunt de infinitis aliis curvis, ut facile deduci potest e nostrâ Miscell. Anal.